# Integrated Structure/Control Law Design by Multilevel Optimization

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**A new approach to integrated structure/control law design based on multilevel optimization is presented. This new approach is applicable to aircraft and spacecraft and allows for the independent design of the structure and control law. Integration of the designs is achieved through use of an upper level coordination problem formulation within the multilevel optimization framework. The method requires the use of structure and control law design sensitivity information. A general multilevel structure/control law design problem formulation is given, and the use of linear quadratic Gaussian control law design and design sensitivity methods within the formulation is illustrated. Results of three simple integrated structure/control law design examples are presented. These results show the capability of structure and control law design tradeoffs to improve controlled system performance within the multilevel approach.**

## Nomenclature

| | |
|---|---|
| $A, B, C, D$ | = state-space coefficient matrices |
| $a_1, a_2$ | = truss bar cross-section areas |
| $c$ | = upper level design criteria vector |
| $E$ | = Young's modulus |
| $f(t)$ | = control force |
| $I$ | = identity matrix |
| $J, K, L$ | = scalar valued objective functions |
| $K$ | = truss stiffness matrix |
| $l$ | = truss bar length |
| $m$ | = concentrated mass |
| $p$ | = design integration parameter vector |
| $Q, R$ | = cost function weighting matrices |
| $r$ | = truss bar midthickness radius |
| $t$ | = truss bar thickness; also time |
| $u$ | = input vector |
| $V, W$ | = noise intensity matrices |
| $w(t)$ | = random disturbance |
| $x$ | = state vector |
| $y$ | = output vector |
| $z_1, z_2$ | = concentrated mass displacements |
| $\alpha, \beta, \chi$ | = scalar design variables |
| $\Theta$ | = control force application angle |
| $\rho, \varphi$ | = cost function weighting matrix scale factors |
| $\nu, \omega$ | = noise intensity scale factors |

*Operators*

| | |
|---|---|
| $E$ | = expected value |
| † | = matrix pseudo-inverse |

*Subscripts*

| | |
|---|---|
| $a$ | = actual |
| $d$ | = desired |
| $m$ | = measured |
| $n$ | = new |
| $o$ | = old |

*Notation*

| | |
|---|---|
| * | = optimal value |
| ‾ | = nominal value |
| $\dot{}, \ddot{}$ | = first and second time derivatives, respectively |
| $\Delta$ | = change |

## Introduction

ONE approach to the design of complex modern aerospace vehicles is to combine high performance, well-designed subsystems and components into a single vehicle that will then be capable of performing the desired mission. For example, both the structure and control system of a large modern spacecraft may be individually designed using sophisticated design methods, iteration, analysis, simulation, and testing to ensure satisfaction of the respective design requirements and objectives. However, integrating these individual components into a single vehicle does not necessarily guarantee good vehicle performance or, in particular, good vehicle dynamics. The reality is that even a well-designed control system may interact with the primary structure in such a way as to cause unacceptably poor or even unstable motions of the vehicle during routine operations. In fact, adverse control/structure interaction problems are considered very likely for future spacecraft like the proposed U.S. space station and many others that may have large solar arrays, flexible antennas, robotic arms, and sensors with strict pointing requirements. Moreover, the problem of control/structure interaction is not limited to spacecraft. Both the F-16 and F-18 fighter aircraft exhibited dynamic control/structure interaction problems in flight, requiring expensive and time-consuming flight control systems redesigns.

As a result of these real and predicted control/structure interaction problems, much research in recent years has been devoted to the development of integrated control/structure design methods. A representative sampling of optimization-based methods is given in Refs. 1-7. In general, these methods

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seek to reduce, eliminate, or perhaps even take advantage of the control/structure interactions in order to meet the total vehicle design performance and stability requirements. This is accomplished by accounting for the interactions within the framework of the methodology and then selecting or modifying the control and/or structural design to meet the integrated design requirements using an optimization algorithm. These methods can be classified further by their approach to the problem as either simultaneous or sequential.[1]

In the simultaneous methods, the control law design and the structural design are directly combined into a single problem. Both the control and structural design variables are then selected to satisfy the integrated design objective, which is usually some combination of the structural and control design objectives. With this approach, the design problem may be of high order since the design mathematical model must include not only the dynamics of both the control system and the structure, but also the combined constraint and design variable sets. Examples of simultaneous methods include Haftka et al.,[2] Salama et al.,[3] Hale,[4] and Miller et al.[5]

The sequential methods of Messac et al.[6] and Khot et al.[7] solve the integrated control/structure design problem by first performing a structural (or control) design followed sequentially by a control (or structure) design. The process is then repeated iteratively until a satisfactory integrated solution is found. Although these methods retain the original sizing of the structure and control law design problems, their main drawback is that the integrated solution is dependent on the sequential ordering of the control and structure design solutions.

In this paper, dynamic vehicle response prediction methods and a well-known control law design method are incorporated into an existing general framework for multidisciplinary design[8,9] in order to solve the integrated control/structure design problem. This framework provides an alternative to both the simultaneous and sequential integrated control/structure design methods. The framework is based on formal design problem decomposition methods,[10,11] sensitivity of optimal solution concepts,[12-14] and multilevel optimization techniques.[8,9,15,16] Development of an integrated control/structure design method within this framework allows for the independent solution of both the structure and control law design problems. Integration of the independently obtained control and structural designs is achieved by formulation and solution of a higher level design coordination problem using the multilevel optimization methods.

The paper is organized as follows. A tutorial explanation of multilevel optimization theory and sensitivity analysis requirements is given first. This is followed by a description of the proposed multilevel integrated control/structure design algorithm. Three control/structure design case studies are then presented for a two-bar truss example problem to illustrate the methodology. The first design case was to improve the closed loop system stability robustness using control law and structural design freedoms, whereas the second and third cases were formulated to expressly examine structure and control law design tradeoffs.

## Multilevel Optimization Theory

A theory of multilevel optimization for large-scale engineering design problems has been developed in recent years.[8,9] This theory is based on hierarchical problem decomposition methods[8-11] and the use of the sensitivity of optimization problem solutions to fixed problem parameters.[12-14]

The theory is explained here by use of a conceptual multilevel optimization problem as shown in Fig. 1. The conceptual problem, which is a quadratic programming problem, is to minimize the performance index $J$ by selection of the design variable $\alpha$ at the upper level and to minimize the performance indices $K$ and $L$ by selection of the design variables $\beta$ and $\chi$, respectively, at the lower level. Note that, in this example, $K$

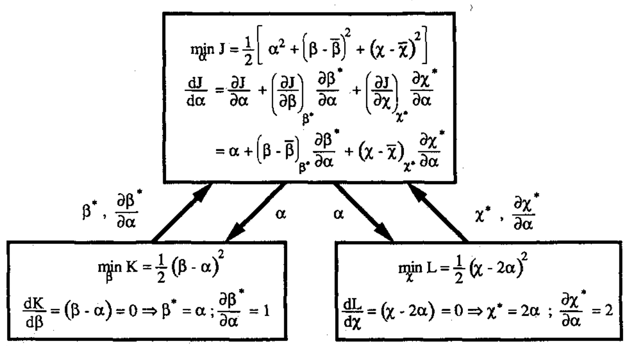



**Fig. 1 Conceptual multilevel optimization problem.**

and $L$ are functions of $\alpha$ and $\beta$ and $\alpha$ and $\chi$, respectively, whereas $J$ is a function of $\alpha$, $\beta$, and $\chi$.

The basic idea behind multilevel optimization is to treat the design variable $\alpha$ as a fixed constant during the minimization of $K$ and $L$. With $\alpha$ fixed, the minimizations of $K$ and $L$ are decoupled and are performed independently. Once the design variables $\beta = \beta^*$ and $\chi = \chi^*$ that minimize $K$ and $L$ are known, the performance index $J$ of the upper level minimization can be evaluated and the gradient of $J$ with respect to $\alpha$ computed.

Clearly, the $\beta = \beta^*$ and $\chi = \chi^*$ that minimize $K$ and $L$, respectively, are functions of the design variable $\alpha$. The evaluation of $J$ at the upper level is for the fixed value of $\alpha$ and $\beta = \beta^*$ and $\chi = \chi^*$. Thus, proper calculation of the gradient of $J$ with respect to $\alpha$ must also include the effects of the changes in $\beta^*$ and $\chi^*$ with respect to changes in $\alpha$. This gradient of $J$ with respect to $\alpha$ can of course be calculated by a direct finite difference method at the upper level. In this case, $J$ is evaluated for small perturbations in $\alpha$, where the minimizations of $K$ and $L$ for $\beta^*$ and $\chi^*$ are repeated for each perturbation. The necessary gradient is then obtained by finite differencing of the perturbed values of $J$.

Another means to calculate the gradient of $J$ with respect to $\alpha$ is to use a chain rule approach. This results in a calculation for the gradient of $J$ at the upper level, which directly involves terms that are the gradients of $\beta^*$ and $\chi^*$ with respect to $\alpha$. In this approach, the gradients of $\beta^*$ and $\chi^*$ with respect to $\alpha$ are calculated during the lower level minimizations of $K$ and $L$ and are passed to the upper level along with the optimized design variables $\beta^*$ and $\chi^*$, as illustrated in Fig. 1. This can simplify the overall multilevel optimization algorithm because the lower level minimizations do not have to be repeated for every perturbation of $\alpha$ at the upper level. (Multiple solutions of the lower level optimization problems may still be required, however, if the gradients of $\beta^*$ or $\chi^*$ with respect to $\alpha$ are calculated by a finite difference method, but they are independent of upper level perturbations of $\alpha$.)

In some cases, analytical expressions for the gradients of $\beta^*$ and $\chi^*$ with respect to $\alpha$ can be derived from the necessary condition of optimality[13-15] for the minimization of $K$ and $L$. This is the case shown in Fig. 1. These expressions are evaluated once for $\beta = \beta^*$ and $\chi = \chi^*$, and the gradient information is passed on to the upper level. Multiple solutions of the lower level optimization problems about a fixed value of the design variable $\alpha$ are eliminated by use of these analytical gradient expressions.

A geometrical interpretation of the sensitivity of optimized solutions is shown in Fig. 2a. In this figure, $K(\beta,\alpha)$ represents the lower level performance index to be minimized by selection of the design variable $\beta$, and $\alpha$ is the parameter with a fixed nominal value for a solution of the optimization problem. A locus of optimal solutions as a function of the value of $\alpha$ is shown in the figure labeled as $K^*(\alpha)$.

The sensitivity derivative of an optimal solution with respect to $\alpha$ gives the slope and direction of the locus of optimized solution at the parameter value corresponding to the optimal solution. This derivative has a component in both the $\alpha$ and $\beta$ directions, a shown in Figs. 2b and 2c, respectively. The

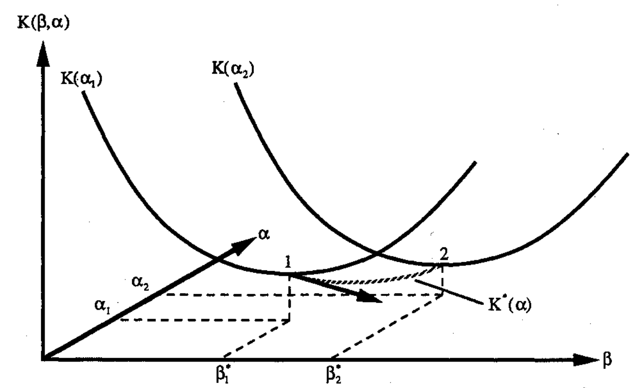



**Fig. 2a Geometrical interpretation of sensitivity of optimum concept.**

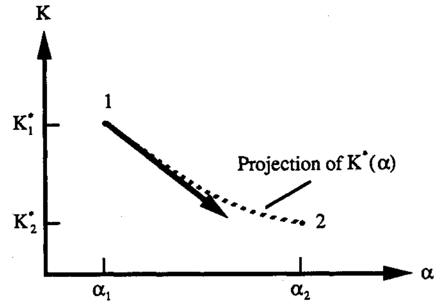



**Fig. 2b $K$, $\alpha$ plane projection.**

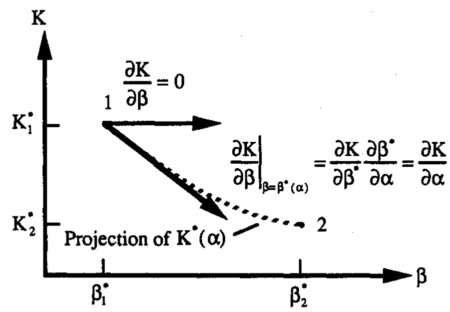



**Fig. 2c $K$, $\beta$ plane projection.**

projection of the sensitivity derivative onto the $K,\alpha$ plane gives the change in the optimized performance index due to changes in the fixed value of the parameter $\alpha$. The projection of the sensitivity derivative into the $K,\beta$ plane gives the change in the optimized performance index, which is due to the change in the (optimal) $\beta$. It would appear that this projection violates the necessary condition of the optimization problem since the derivative of $K$ with respect to arbitrary variations in $\beta$ must be zero at the optimum. In other words, the derivative $\partial K/\partial\beta$ must be zero along a line parallel to the $\beta$ axis. This is, in fact, the case here also. The nonzero projection in the $K,\beta$ plane is caused by the constraint that the sensitivity derivative be calculated for $\beta = \beta^*(\alpha)$. That is, the $K,\beta$ projection is really $\partial K/\partial\alpha$ mapped from the $K,\alpha$ plane by $\beta = \beta^*(\alpha)$.

The sensitivity of the optimized performance index is an incomplete measure of the change in optimal solutions since there is no information on the charge in the design variables due to parameter variations. This is particularly evident in the simple example of Fig. 1, where the lower level minimization of $K$ is considered. In that problem, the optimized performance index is $K^* = 0$ for all values of the parameter $\alpha$, but the optimizing design variable is $\beta^* = \alpha$. Use of the sensitivity of $K^*$ with respect to the parameter $\alpha$ would not yield any useable information for the upper level optimization since that sensitivity is identically zero for all values of $\alpha$. In the example, then, the square difference of $\beta$ from a desired value was used as the criterion at the upper level, which is related to the lower level $K$ minimization problem. The introduction and use of additional criteria to measure lower level performance at the upper levels is a requirement of multilevel optimization algorithms.

Several iterations of the conceptual problem of Fig. 1 for the case $\overline{\beta} = \overline{\chi} = 1$ are shown below as a final illustration of multilevel algorithms. Note in particular that, once an $\alpha$ is selected, the lower level solutions for $\beta^*$ and $\chi^*$ are decoupled and are obtained independently of each other. (For the purposes of this illustration, a different scale factor $\eta$ in the computation for $\Delta\alpha$ was used at each step to limit the number of iterations.)

Iteration 1 ($\alpha_1 = 1$ assumed to start)

$$\alpha_1 = 1 \Rightarrow \beta_1^* = 1, \qquad \chi_1^* = 2$$

$$J_1 = 1, \qquad \frac{dJ}{d\alpha} = \alpha_1 + (\beta_1^* - 1)(1) + (\chi_1^* - 1)(2) = 3$$

$$\Delta\alpha_1 = -\eta J_1 \frac{dJ^{-1}}{d\alpha} = (-1)\frac{1}{3} = -\frac{1}{3}, \qquad (\eta = 1)$$

Iteration 2

$$\alpha_2 = \alpha_1 + \Delta\alpha_1 = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow \beta_2^* = \frac{2}{3}, \qquad \chi_2^* = \frac{4}{3}$$

$$J_2 = \frac{1}{3}, \qquad \frac{dJ}{d\alpha} = \alpha_2 + (\beta_2^* - 1)(1) + (\chi_2^* - 1)(2) = 1$$

$$\Delta\alpha_2 = -\eta J_2 \frac{dJ^{-1}}{d\alpha} = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(1) = -\frac{1}{6}, \qquad \left(\eta = \frac{1}{2}\right)$$

Iteration 3

$$\alpha_3 = \alpha_2 + \Delta\alpha_2 = \frac{2}{3} - \frac{1}{6} = \frac{1}{2} \Rightarrow \beta_3^* = \frac{1}{2}, \qquad \chi_3^* = 1$$

$$J_3 = \frac{1}{4}, \quad \frac{dJ}{d\alpha} = \alpha_3 + (\beta_3^* - 1)(1) + (\chi_3^* - 1)(2) = 0 \Rightarrow \alpha^* = \alpha_3 = \frac{1}{2}$$

Analytical solution (obtained by substituting the expressions for $\beta^*$, $\chi^*$, and their derivatives into the gradient of $J$ with respect to $\alpha$ and setting the result equal to zero)

$$\frac{dJ}{d\alpha} = \alpha + (\beta^* - \overline{\beta})\frac{\partial\beta^*}{\partial\alpha} + (\chi^* - \overline{\chi})\frac{\partial\chi^*}{\partial\alpha} = \alpha + (\alpha - \overline{\beta})(1)$$

$$+ (2\alpha - \overline{\chi})(2)$$

$$= 6\alpha - \overline{\beta} - 2\overline{\chi} = 0 \Rightarrow \alpha^* = \frac{1}{6}(\overline{\beta} + 2\overline{\chi}) = \frac{1}{6}(1 + 2) = \frac{1}{2}$$

## Multilevel Structure/Control Law Design Approach

A multilevel optimization-based method can be developed for integrated control/structure design. Specific formulation for such algorithms are, of course, highly problem dependent, however, a general formulation that uses linear quadratic Gaussian (LQG) optimal control law design methods to determine the control law is outlined in the following.

The general multilevel structure/control law design formulation is shown in Fig. 3. In Fig. 3, the structural design and the control law design are independent lower level design

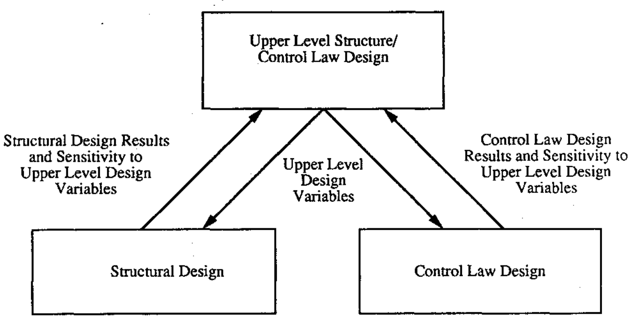


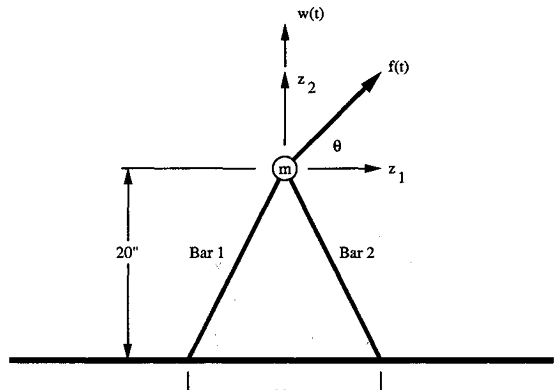

**Fig. 3 General multilevel structure/control law design algorithm.**

problems. These lower level designs are coordinated through the upper level optimization problem. The upper level optimization problem reflects the desired objectives of the integrated structure/control law design. For example, the upper level objective might be to reduce peak transient responses of the controlled system and to reduce the weight of the structure. The actual peak transient responses of the controlled system would come from analysis of the control law design at the lower level, whereas the actual structural weight would come from the lower level structural optimization. These could then be combined as a weighted sum of square errors between the actual and desired values to form a single upper level performance index. The upper level design variables would then be selected to minimize the objective function.

The values of the upper level design variables at any time are treated as fixed parameters for the lower level optimizations. These parameters define either the mathematical model of the structure or dynamic system to be controlled or they define the performance index and/or constraints of the optimization problems, or both. The sensitivities of the optimized lower level solutions to these fixed parameters are computed and used in turn to compute the sensitivity of the related part of the upper level performance index. That is, these final sensitivities are the gradients necessary to complete the top level optimization. In the earlier example, one of the upper level design variables may be a local structural stiffness requirement, which appears as an equality constraint in the lower level structural optimization. The sensitivity of the structural weight to this parameter is computed at the lower level and returned for use in computing the part of the gradient of the upper level performance index that is related to structural weight. Another of the upper level design variables might be a mean square weight on control effort, which would appear directly in the performance index of the lower level LQG control law design problem. The sensitivity of the optimized LQG control law with respect to this parameter would then be used to compute the sensitivity of the peak transient response of the controlled system, as required to complete the upper level optimization.

For the purposes of the current development, the use of existing nonlinear programming-based structural optimization and design sensitivity analysis methods is assumed. These methods may themselves be multilevel optimization algorithms, such as those of Refs. 15 and 16.

The use of LQG optimal control law design methods is also assumed in the current method. Expressions for the sensitivity of controlled system time, frequency, and stochastic responses in terms of state-space coefficient sensitivity matrices are available from the literature. The sensitivity of the optimized LQG control law to fixed parameters must be known to compute the necessary state-space coefficient matrices. Analytical expressions for the sensitivity of the LQG gain matrices to fixed problem parameters have been derived elsewhere from the necessary conditions of optimality.[17] The analytical expressions for the LQG sensitivity as well as analytical sensitivity equations for frequency responses, time responses, covariance responses, eigenvalues, and singular values are summarized in Ref. 18.




Mass = m = 2
Area Bar 1 = $a_1$ = 500
Area Bar 2 = $a_2$ = 1000
Length Bar 1 = Length Bar 2 = l = 22.36"
Force Application Angle = Θ = 45°
Young's Modulus E = 1

**Fig. 4 Two-bar truss geometry and data.**

## Two-Bar Truss Example

### Description

A two-bar truss control/structure design problem was taken directly from the literature[7] and used to validate the multilevel integrated control/structure design method. The results of design case studies with this problem are given here to illustrate the proposed integrated control/structure methodology. Other applications of the methodology to problems of higher complexity have recently been completed and further validate the current approach.[19]

The nominal problem definition and truss structure geometry is shown in Fig. 4. The basic design problem is to find a feedback control law for the external force $f(t)$ and to define the structural element geometry such that the lateral and vertical motions of the concentrated mass $m$ are minimized when the truss is subject to an external disturbance $w(t)$. Because the nominal truss bar geometries are not the same, the lateral and vertical motions of the mass are coupled. Only the extensional stiffness of the truss bars is considered. The hierarchical problem decomposition and the specific criteria and design variables used in the problem are discussed in following sections.

State-space equations of motion for the truss are given by

$$\begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \ddot{z}_1 \\ \ddot{z}_2 \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -\frac{1}{m}K & -0.01\frac{1}{m}K \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \cos\Theta \\ \sin\Theta \end{bmatrix} f(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w(t)$$

$$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix}$$

or

$$\dot{x} = Ax + Bu + Dw$$

$$y = Cx$$

**Table 1 Case 1 design criteria**

| Criteria number | Original value | Desired value | Description |
|---|---|---|---|
| 1 | −0.506 | −0.531 | Real parts of closed-Loop structural mode pair 1 |
| 2 | −1.452 | −1.524 | Real parts of closed-Loop strucutral mode pair 2 |
| 3 | 0.834 | 0.900 | Return difference singular value at 1.00 rad/s |
| 4 | 0.832 | 0.900 | Return difference singular value at 3.35 rad/s |
| 5 | 0.859 | 0.900 | Return difference singular value at 5.99 rad/sec |
| 6 | 1.263 | 0.900 | Loop transfer magnitude at 5.21 rad/s |
| 7 | 1.251 | 0.900 | Loop transfer magnitude at 5.37 rad/s |

**Table 2 Case 1 original design and sensitivity data for first iteration**

| Criteria number | Original value | $\frac{\partial c_i}{\partial p_1}$ | $\frac{\partial c_i}{\partial p_2}$ | $\frac{\partial c_i}{\partial p_3}$ | $\frac{\partial c_i}{\partial p_4}$ |
|---|---|---|---|---|---|
| 1 | −0.506 | −0.210 | 0.000 | $-2.923\times 10^{-4}$ | −0.762 |
| 2 | −1.452 | −0.542 | 0.000 | $1.031\times 10^{-4}$ | 0.851 |
| 3 | 0.834 | $5.148\times 10^{-3}$ | 4.274 | $2.874\times 10^{-4}$ | $-3.891\times 10^{-2}$ |
| 4 | 0.832 | $-1.178\times 10^{-2}$ | 4.441 | $3.652\times 10^{-4}$ | $-3.922\times 10^{-2}$ |
| 5 | 0.859 | $-4.637\times 10^{-2}$ | 1.962 | $1.416\times 10^{-4}$ | $-8.860\times 10^{-2}$ |
| 6 | 1.263 | 0.451 | $-2.263\times 10^{1}$ | $-4.245\times 10^{-3}$ | 1.874 |
| 7 | 1.251 | 0.461 | $-2.393\times 10^{1}$ | $4.410\times 10^{-3}$ | 1.820 |

where the definitions of $x$, $y$, $u$, $w$, $A$, $B$, $C$, and $D$ follow from the state equations, and the stiffness matrix $K$ for the truss is

$$K = \frac{E}{5l}\begin{bmatrix} a_1 + a_2 & 2(a_1 - a_2) \\ 2(a_1 - a_2) & 4(a_1 + a_2) \end{bmatrix}$$

Note that structural damping of 0.01 has been assumed in this model.

The bar cross-sectional areas and the Young's modulus $E$ are scaled in this example as in Ref. 7 such that the natural frequencies of vibration are consistent with an actual structure of this size and concentrated mass. With these scalings, the material mass density ($= 2.59 \times 10^{-6}$ lb-s$^2$/in.$^4$) is such that the mass of the bars is negligible compared to the concentrated mass for the vibration equations of motion. The bar cross-sectional areas are, however, a direct measure of the actual truss structure weight, which is to be reduced as part of the integrated control/structure design objective.

### Lower Level Control Law Design

An LQG optimal control law problem was formulated for the lower level control law design in the two-bar truss example. A cost function

$$J = \lim_{T\to\infty} \frac{1}{T} E \int_0^T (y^T Q y + u^T R u)\, d\tau$$

was assumed, where $E$ denotes expected value and the weighting matrices $Q$ and $R$ are $Q = \varphi I$ and $R = \rho$, where the nominal values of $\varphi$ and $\rho$ are 1.0 and 0.01, respectively. Noisy measured feedback signals were assumed as $y_m = y + v$, where the noise $v(t)$ is a zero mean, white noise with intensity matrix $V = \nu I$, with the nominal value of $\nu = 0.01$. The disturbance input $w(t)$ was assumed to be zero mean, white noise as well, with intensity $W = \omega$, $\omega = 1.0$ nominally. The noises $w$ and $v$ were assumed to be uncorrelated.

### Lower Level Structural Design

A closed-form structural design problem was defined for the two-bar truss example problems. A closed-form problem was used to simplify the example problem while simulating an actual structural design. This problem was to find the thickness of truss bar 1 assuming the bar was a hollow circular tube with a specified cross-sectional area. In this case, the thickness $t$ of the tube is just $t = a_1/2\pi r$, where $r$ is the midthickness radius of the tube.

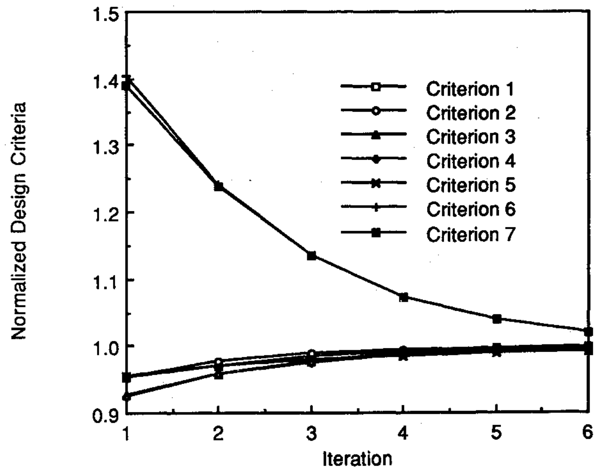



**Fig. 5 Case 1 design criteria iteration history.**

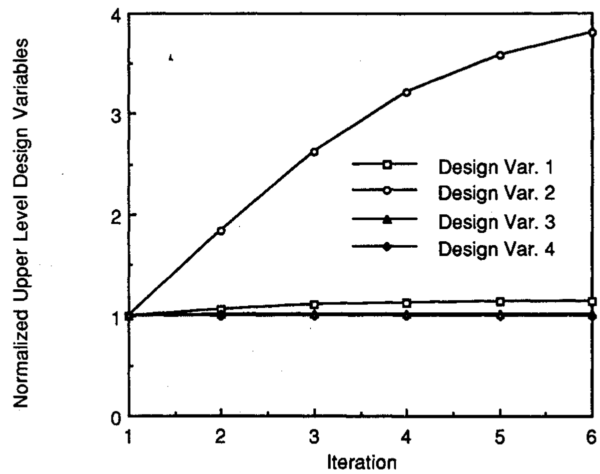



**Fig. 6 Case 1 upper level design variable history.**

### Upper Level Integration Problem

Four parameters were selected for use as design integration parameters and, thus, as design variables at the top level of the multilevel structure/control law design examples. These are the scale factor $\varphi$ defining the controlled output weighting matrix $Q$ in the LQG cost function (parameter 1, $p_1$), the scale factor $\nu$ defining the LQG noise intensity $V$ ($p_2$), the cross-sectional area $a_1$ of truss bar 1 ($p_3$), and the control force application angle $\Theta$ ($p_4$). These parameters were chosen to provide the top-level problem with influence over the lower level control design (parameters 1 and 2), influence over the structural design (parameter 3), and a means of trading structure vs control effort (parameter 4)

The upper level objective function for three design cases was written to minimize the sum of the square difference between desired and actual values of certain design criteria as

$$S = \sum_{i=1}^{n} (c_{d_i} - c_{a_i})^2$$

The specific design criteria for each design case are given in the following.

#### Design Case 1

Design case 1 was formulated to improve closed-looped stability robustness of the integrated system. Seven measures of controlled system stability robustness were used in the top-level objective function. These criteria included increasing the damping of the two structural modes by moving their eigenvalues to the left in the complex plane, raising the minimum singular value of the return difference at three discrete frequencies, and reducing the magnitude of the loop transfer function at two discrete frequencies.

The initial values of the seven criteria and their sensitivity to each of the four top-level design variables were computed for the design variables at their initial, nominal values. Desired values of these seven design criteria were then selected arbitrarily based on the initial results, and the top-level objective function was formulated. Table 1 summarized the initial and desired values of the seven criteria. The sensitivity results for the seven criteria relative to the four upper level design variables are summarized in Table 2. The sensitivity data was arranged accordingly in a gradient matrix and an incremental change in the four design variables was computed as

$$\Delta p = \frac{\partial c^{\dagger}}{\partial p} (c_d - c_a)$$

where $p$ is a vector of the four upper level design variables. New values of the four design variables were then selected as

$$p_n = p_o + \tfrac{1}{2}\Delta p$$

where the subscripts $n$ and $o$ denote new and old and the factor of ½ was selected to reduce the effects of the linearization error. This process was repeated for five iterations.

The design results are shown in Figs. 5 and 6. In Fig. 5, the design criteria iteration history is shown with the results normalized by the desired value, so that satisfaction of a criterion occurs for a value of 1.0. All seven criteria are moving toward satisfaction with each iteration. Figure 6 shows the history of the upper level design variables, normalized by their initial (starting) value. There is a large increase in the $V$ matrix scale factor $\nu$, which is effectively tuning the Kalman filter design, a slight increase in the $Q$ matrix scale factor $\varphi$, which is tuning the regulator design, a slight increase in structural weight (increase in $a_1$); and a decrease in the control force application angle $\Theta$. These results indicate that combination of structure and control law design changes can be used to improve the overall stability robustness of a controlled system.

#### Design Case 2

The second design case was formulated to tradeoff structure and control design objectives. The upper level design objective was written to improve six integrated design criteria. The criteria included the mean-square response and control energy due to a random disturbance input $w(t)$, the static controlled system structural deflections due to a static load, and the structural weight (directly related to the cross-sectional area of bar 1). The objective was formulated as in case 1, with the desired values of the six criteria arbitrarily set to 95% of the initial values. The criteria are further described in Table 3.

The normalized results are shown in Figs. 7 and 8. Both the control force and the structural weight in Fig. 7 were reduced toward their desired values, whereas the other four criteria were actually increased. Looking at the upper level design variable iteration history in Fig. 8, the cross-sectional area $a_1$ was reduced to 95% of the original value, reflecting the direct influence of its magnitude in the upper level objective. The control force application angle $\Theta$ was increased, the $Q$ matrix

**Table 3 Case 2 and case 3 design criteria**

| Criteria number | Original value | Desired value | Description |
|---|---|---|---|
| 1 | $1.194\times10^{-2}$ | $1.135\times10^{-2}$ | Mean-square deflection in $x$ direction |
| 2 | $3.125\times10^{-2}$ | $2.968\times10^{-2}$ | Mean-square deflection in $y$ direction |
| 3 | 0.646 | 0.614 | Mean-square control force |
| 4 | $3.629\times10^{-2}$ | $3.447\times10^{-2}$ | Steady-state $x$ deflection to step load |
| 5 | $4.490\times10^{-2}$ | $4.266\times10^{-2}$ | Steady-state $y$ deflection to step load |
| 6 | $5.000\times10^{2}$ | $4.750\times10^{2}$ | Bar 1 cross-sectional area (truss weight) |

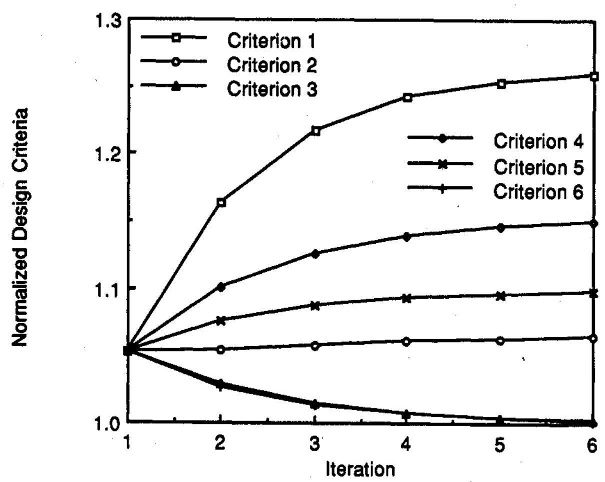



**Fig. 7 Case 2 design criteria iteration history.**

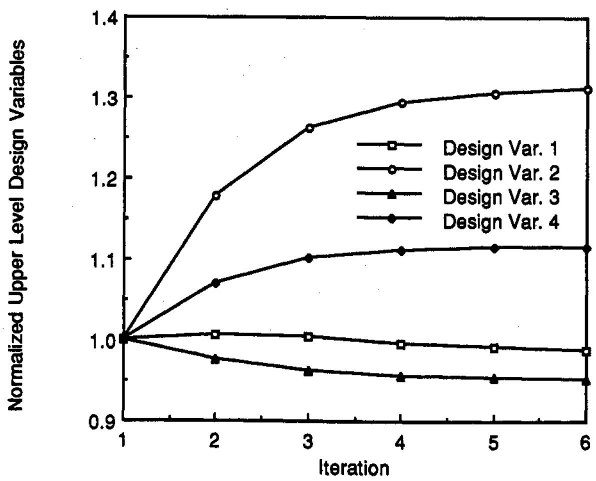



**Fig. 8 Case 2 upper level design variable history.**

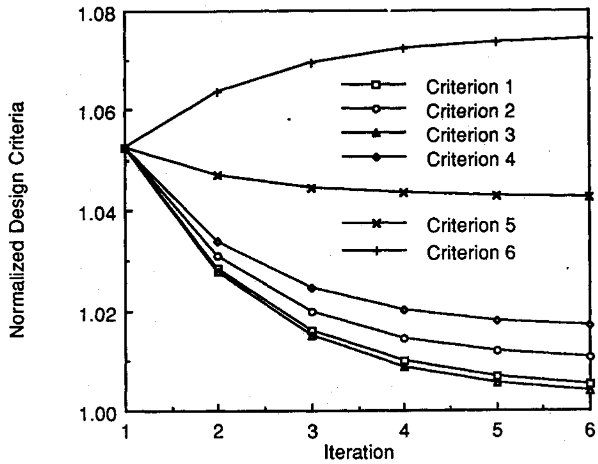



**Fig. 9 Case 3 design criteria iteration history.**

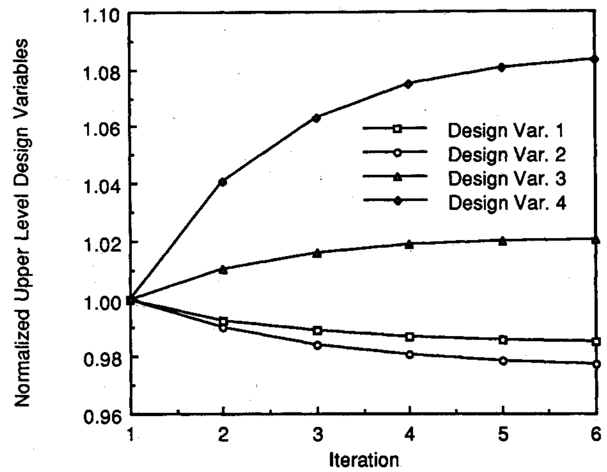



**Fig. 10 Case 3 upper level design variable history.**

scale factor $\varphi$ was slightly decreased, and the $V$ matrix scale factor $\nu$ was increased 31%. The conclusion here is that the actual magnitude of the cross-sectional area dominates the design.

**Design Case 3**

Design case 3 was the same as design case 2 except that the upper level design criteria were normalized by their initial values before the upper level objective function was formulated. The results for this case show that five of the six criteria are improved (Fig. 9), the exception now being the structural weight which is increasing. The design parameter history in Fig. 10 shows the 2% increase in bar cross-sectional area $a_1$ and also an 8-9% increase in force application angle $\Theta$. The results show the dependence of the design on the magnitudes of the design criteria relative to one another. They also indicate that small increases in structural weight may be worthwhile in terms of controlled system performance. This is intuitively satisfying since decreases in structural deformations are often associated with increased structural weight.

## Conclusions

A new approach to the integrated structure/control law design problem of aerospace vehicles has been presented. This approach uses multilevel optimization techniques that are based on the theories of hierarchical problem decompositions, optimization, and sensitivity of optimum solutions. Unlike existing sequential or simultaneous methods, the structure and control law designs are obtained independently. Integration of the structure and control law designs is achieved through an optimization problem formulation in which the dependent disciplinary designs are coordinated at an upper level. Multilevel optimization has been explained by use of a simple conceptual example and a general multilevel structure/control law design algorithm outlined. Results for three integrated structure/control law designs of a two-bar truss example problem are presented. These results illustrate the multilevel design method and show the potential tradeoffs that are possible between the structure and control law designs. The examples point out once more the importance of the relative magnitudes of the design criteria to the final results.

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